# Resonances while surmounting a fluctuating barrier

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(Received 7 September 1999)

Electronic analog experiments on escape over a fluctuating potential barrier are performed for the case when the fluctuations are caused by Ornstein-Uhlenbeck noise (OUN). In its dependence on the relation between the two OUN parameters (the correlation time $\tau$ and noise strength $Q$) the nonmonotonic variation of the mean escape time $\mathcal{T}$ as a function of $\tau$ can exhibit either a minimum (resonant activation), or a maximum (inhibition of activation), or both these effects. The possible resonant nature of these features is discussed. We claim that $\mathcal{T}$ is not a good quantity to describe the resonancelike character of the problem. Independently of the specific relation between the OUN parameters, the resonance manifests itself as a maximal lowering of the potential barrier during the escape event, and it appears for $\tau$ of the order of the relaxation time toward the metastable state.

PACS number(s): 05.40.−a, 82.20.Mj, 02.50.Ey

## I. INTRODUCTION

In classical systems the escape of a particle from a local potential minimum over a potential barrier is possible due to interaction with a thermal bath. Independently of the specific measure used to characterize the duration of the escape process, the average time $\mathcal{T}$ spent waiting for a successful jump depends on the height of the barrier $\Delta U$ and the temperature $T$ of the bath through the Arrhenius formula: $\mathcal{T} \sim \exp(\Delta U/kT)$. Lately, it has been shown that this time may be significantly reduced or prolonged by correlated stochastic perturbation of the barrier. In the context of recent interest in resonancelike phenomena in noisy dynamics, it is natural to look for a relationship between $\mathcal{T}$ and the characteristic time of the perturbation given by its correlation time $\tau$. In 1992, studying a triangle barrier switched randomly between the two possible configurations, Doering and Gadoua [1] discovered that $\mathcal{T}(\tau)$ may exhibit a minimum for $\tau$ of the order of the escape time over the lower possible configuration of the barrier. They therefore called this effect *resonant activation* (RA). Later it was shown [2–5] that this resonant relation between the time scales of the system is characteristic of those cases where the potential barrier is disturbed by dichotomic noise (DN). If a Gaussian correlated noise, i.e., an Ornstein-Uhlenbeck noise (OUN), is applied the resonant minimum of $\mathcal{T}$ occurs when $\tau$ is of the order of the relaxation time toward the metastable state [6,3–5,7].

In [8] one of the present authors concluded that the opposite effect—the occurrence of a maximum in the $\tau$ dependence of $\mathcal{T}$—can be observed, too (see also [9]), although its possible resonance origin remained unknown. This was called *inhibition of activation* (IA). Moreover, considering the exact formulas for the mean first passage time (MFPT) over a barrier disturbed by OUN, it was inferred that a nonmonotonic form for $\mathcal{T}(\tau)$ is generic and conditioned by the relationship between $\tau$ and the intensity of the noise $Q$. In particular, RA occurs if $Q$ is a linear function of $\tau$, i.e., if the variation of the noise is constant. On the other hand, if $Q$ is $\tau$ independent one could expect IA to occur. These conclusions agree with the theoretical [10,6,4,7], numerical [4,7], and experimental [6,5] findings of other authors.

In [8] a more general class of noises, with $Q$ being a more complicated function of $\tau$, was also considered and some universal criteria for the appearance of RA and IA were found. In order to verify them we have performed experiments on analog electronic circuits, the results of which are presented and discussed below (Sec. IV). The conclusions of this study allow us to verify the reason for the appearance of the extremes of $\mathcal{T}(\tau)$ and, if they are of a resonance nature, which quantities are in resonance (Sec. V). But we start (Sec. II) by presenting a brief resume of the previous findings [8] and then (Sec. III) specifying the model investigated and discussing some experimental details.

## II. THEORETICAL PREDICTIONS

Let us consider the overdamped one-dimensional motion of a particle in a bistable potential $U(x)$ in the presence of a heat bath. In our study the potential is also modulated in time by a stochastic perturbation which, for simplicity, does not alter the positions of extremes of the total potential. The dynamics of the particle is governed by the following Langevin equation:

$$\frac{dx}{dt} = -U'(x) - V'(x)z(t) + \xi(t), \tag{1}$$

where thermal fluctuations are represented by Gaussian white noise $\xi(t)$ of zero mean and correlation function

$$\langle \xi(t)\xi(t')\rangle = 2q\,\delta(t-t'). \tag{2}$$

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The intensity $q$ of the noise is linearly proportional to the bath temperature. The fluctuating part of the potential is driven by OUN,

$$\frac{dz}{dt}=-\frac{1}{\tau}z+\frac{\sqrt{2Q}}{\tau}\,\eta(t), \tag{3}$$

where $\eta(t)$ is another Gaussian white noise independent of $\xi(t)$, of zero mean and correlation function $\langle \eta(t)\eta(t')\rangle = \delta(t-t')$. The relation between the two parameters of this noise, namely, its strength $Q$ and its correlation time $\tau$, appears to be crucial for the appearance of RA or IA, so, in general, $Q$ should be considered as a function of $\tau$. As was shown in [8], the general tendencies in the dependence of MFPT on $\tau$ can be found by analyzing the problem in the limits of very fast ($\tau\rightarrow 0$) and very slow ($\tau\rightarrow\infty$) barrier fluctuations, only. We do not consider here the general form of $Q(\tau)$ treated in [8]. To discuss all the main features of the escape problem it is enough to assume that for any $\tau$ the noise strength $Q$ has the following form:

$$Q(\tau)=Q_0\tau^{\alpha}, \quad 0<Q_0<\infty, \quad 0\leqslant\alpha\leqslant 1. \tag{4}$$

In the limit $\tau\rightarrow 0$ for $\alpha=0$ the noise $z(t)$ becomes white, while for $\alpha>0$ the noise intensity $Q$ goes to zero, so $z(t)$ vanishes. The opposite limit $\tau\rightarrow\infty$ can be discussed in a similar way. However, a better quantity to use in the discussion below is the noise variance $D$ given as

$$D=Q/\tau. \tag{5}$$

Thus, if $\alpha<1$ the noise $z(t)$ disappears since $D\rightarrow 0$. However, if $\alpha=1$ then $D\rightarrow Q_0$, so we acquire an ensemble of static potentials spread according to a Gaussian distribution with the variance $D$. Let us mention that the cases of $\alpha=0$ and $\alpha=1$ are the commonly used variants of OUN: constant-strength noise (CSN) $[Q(\tau)=Q_0]$ and constant-variance noise (CVN) $[Q(\tau)=\tau Q_0]$, respectively.

The main conclusions of [8] were as follows. First, independently of the specific form of $Q$, we have

$$\mathcal{T}_0\leqslant\mathcal{T}_s\leqslant\mathcal{T}_\infty, \tag{6}$$

where the indices 0 and $\infty$ refer to the appropriate limit of $\tau$, and $\mathcal{T}_s$ is the MFPT for an unperturbed (static) barrier. It is obvious that the equalities relate to the cases of vanishing noise mentioned above. The analysis of the leading order corrections of $\mathcal{T}$ for finite $\tau$ shows that the inequalities (6) are also fulfilled in some proximity of these limits. Thus for small $\tau$ when $\alpha>0$ the escape time $\mathcal{T}$ always decreases with increasing $\tau$, while for large $\tau$ and $\alpha<1$ a decrease of $\tau$ causes an increase of $\mathcal{T}$. This assures that a minimum or a maximum appears, respectively. The explanation of such behavior is very simple: if $z(t)$ vanishes in a given limit then, for a finite value of $\tau$, it does exist and causes an effect similar to that of a nonvanishing noise. When $z(t)$ does not vanish, if we do not deal with certain specific forms of $U(x)$ and $V(x)$, then the escape time $\mathcal{T}$ always increases in the region of small $\tau$, and so RA does not appear. On the other hand, since in this case $\alpha=0$, IA should be observed. A similar argument applies to the case of CVN ($\alpha=1$). The existence of RA is a generic property, while in the large-$\tau$ region $\mathcal{T}$ grows monotonically and IA is absent.

## III. SYSTEM

The utility of the electronic analog technique for modeling stochastic dynamics has been demonstrated in many cases (e.g., see the recent review [11]). However, the problem of escape over a fluctuating barrier seems to have been investigated in this way only in experiments of the Perugia-Camerino group [6,5]. The authors considered two kinds of colored noises: DN and OUN, both in two variants: CSN and CVN. In our research we do not deal with DN; however, we consider a much wider class of OUN's. On the other hand the potential $V(x)$ used in [6,5] was a simple parabolic one, so it caused a permanent variation of the position of potential minima, leading even to the disappearance of the bistable character of the total potential. Here we use another form of $V(x)$ that avoids these inconsistencies.

The circuit used in our experiments has been based on a standard electronic system simulating Langevin equation with a quartic potential,

$$U(x)=\frac{1}{4}x^4-\frac{1}{2}x^2, \tag{7}$$

with a maximum at $x=0$ and two minima at $\pm 1$. The perturbation has the form

$$V(x)=\begin{cases} U(x) & \text{for } |x|\leqslant 1\\ 0 & \text{for } |x|>1,\end{cases} \tag{8}$$

so it does not alter the positions of potential extremes and the fluctuations affect only the barrier itself.

The system was prepared at random in one of the potential minima ($x=\pm 1$). The time of its first appearance at the top of the barrier $t_{\rm top}$ was then measured. We observed also the value of the colored noise at this moment $z_{\rm top}\equiv -z(t_{\rm top})$, where minus is used for later convenience. At least 2000 jumps from each well were recorded. The symmetry of the system was checked very carefully, so in fact we dealt with the statistics of at least $N=4000$ events. From the data collected, we calculated the MFPT $\mathcal{T}$ and its standard deviation $\Delta\mathcal{T}$, as well as the mean value $Z$ of $z_{\rm top}$ and its standard deviation $\Delta Z$.

The control parameter of the problem, the correlation time of the OUN, was varied within the interval $10^{-2}<\tau<10^3$. The measurements were repeated for five different relations between $Q$ and $\tau$ with $\alpha=0$, 0.25, 0.50, 0.75, and 1.0. The other parameters were kept constant: $q=0.067$ and $Q_0=0.73$. In what follows we use scaled quantities in order to ensure a simple form (1) of the Langevin equation with the potential (7). The time unit of this paper corresponds to 1.02 ms of real time, so the measured value 110 ms of the MFPT over an unperturbed barrier gives $\mathcal{T}_s=107$. Finally, the correlation times of the noise generators were of the order of a few $\mu$s, so effectively we are dealing with white noises.

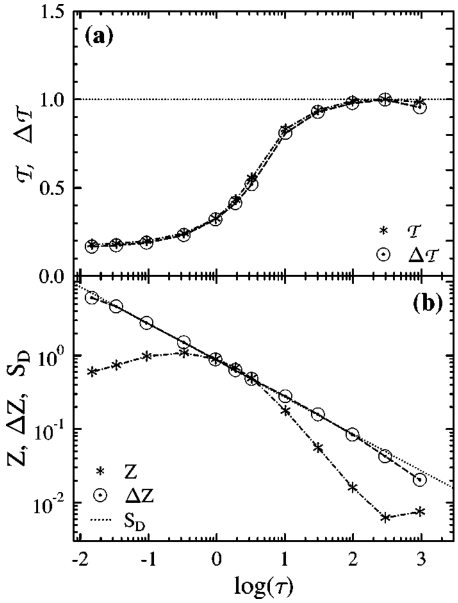


FIG. 1. Experimental data measured for an analog electronic model of Eqs. (1), (3), (4), (7), and (8) vs decimal logarithm of $\tau$. (a) Relative MFPT $\mathcal{T}(\tau)/\mathcal{T}_s$ and its standard deviation $\Delta\mathcal{T}(\tau)/\mathcal{T}_s$ for $\alpha=0.0$. For reference the dotted line indicates the relative MFPT for a static barrier. The values of the other parameters are $q=0.0674$ and $Q_0=0.734$. (b) The mean value $Z(\tau)$ of the colored noise and its standard deviation $\Delta Z(\tau)$ as measured at the moment of crossing the top of the barrier. For comparison the $\tau$ dependence of the noise standard deviation, $S_D=[Q(\tau)/\tau]^{1/2}$, is also displayed (dotted line). In all figures the lines that connect the experimental points are drawn to guide the eye, only.

## IV. EXPERIMENTAL RESULTS

### A. Escape time

The results of the experiments, collected for $\alpha=0$, 0.25, 0.50, 0.75, and 1.0, are summarized in Figs. 1–5, respectively. In the (a) parts of the figures the MFPT and its standard deviation are displayed. In accordance with the theoretical predictions for $\alpha>0$, the escape time $\mathcal{T}(\tau)$ develops a minimum on the small-$\tau$ side. Only for CSN does $\mathcal{T}(\tau)$ increase monotonically in this region. Similarly, for $\alpha<1$, a maximum exists as expected on the large-$\tau$ side of the figure; but a monotonic increase characterizes the case with CVN. We notice, however, that the minima are more clearly defined than the maxima.

The position of the minimum $\tau_{\min}$ depends strongly on $\alpha$. As $\alpha$ decreases the minimum shifts significantly toward smaller values of $\tau$, e.g., $\tau_{\min}$ for $\alpha=0.25$ is about ten times smaller than for $\alpha=1$. Simultaneously, the minimal values of the MFPT $\mathcal{T}_{\min}$ change only slightly, while the width of the minimum increases. These properties of $\mathcal{T}(\tau)$ result simply from the $\tau$ dependence of $Q$. If $\alpha>0$ the OUN $z(t)$ vanishes as $\tau\rightarrow 0$ and $\mathcal{T}(0)=\mathcal{T}_s$ [8]. On the other hand, as $\alpha$ decreases colored noise approaches CSN, which does not disappear in

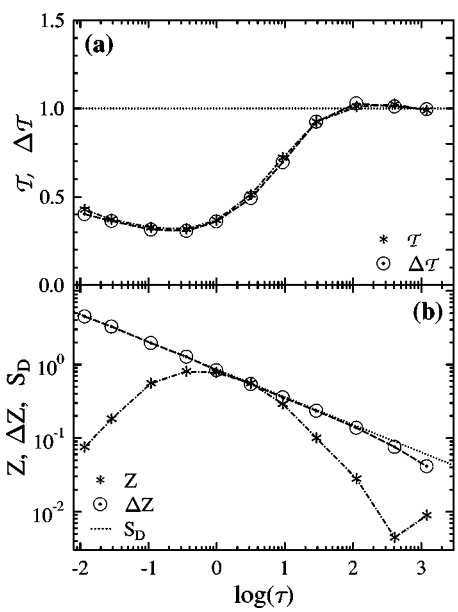


FIG. 2. The same as Fig. 1 but for $\alpha=0.25$.

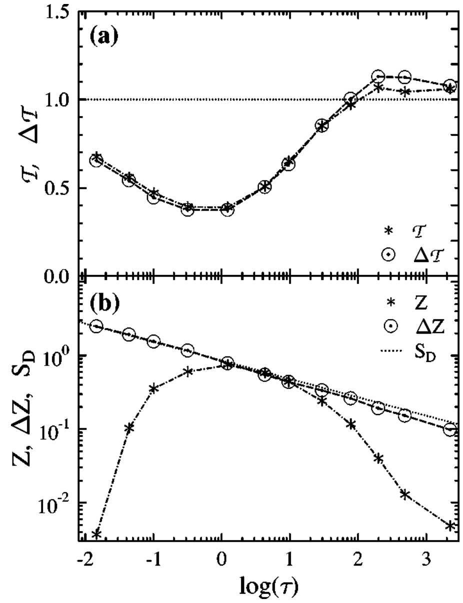


FIG. 3. The same as Fig. 1 but for $\alpha=0.5$.

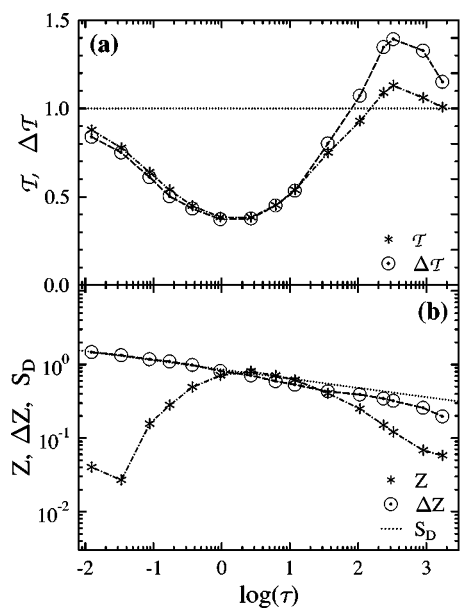


FIG. 4. The same as Fig. 1 but for $\alpha=0.75$.

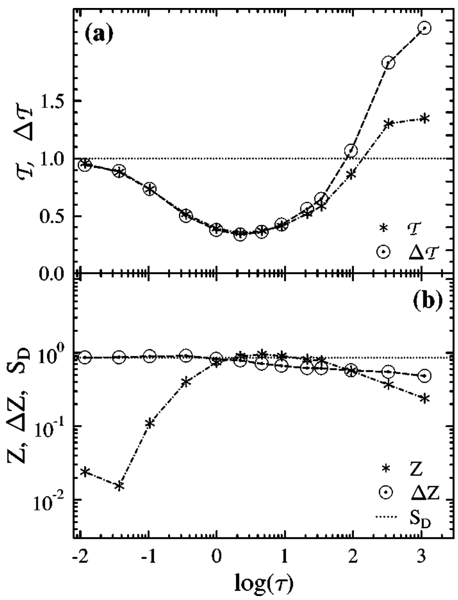


FIG. 5. The same as Fig. 1 but for $\alpha$=1.0.

this limit, and for which $\mathcal{T}(0)<\mathcal{T}_s$. Thus the nonmonotonic curve $\mathcal{T}(\tau)$ for $\alpha>0$ converges to the monotonic one with $\alpha=0$, which means that $\tau_{\min}$ shifts toward zero. Since the position of the minimum depends on the rate of variation of $Q$ with $\tau$ and may be located within an infinite (on a logarithmic scale) interval, we cannot treat the appearance of a minimum of $\mathcal{T}(0)=\mathcal{T}_s$ as the signature of a resonance between the noise $z(t)$ and any characteristic time of the system.

Although the maxima are not so clear as the minima, one can notice a very similar relationship between the value of $\alpha$ and the location of a maximum $\tau_{\max}$: as $\alpha$ increases $\tau_{\max}$ also increases. This is a consequence of the vanishing of $z(t)$ in the limit $\tau\to\infty$ for $\alpha<1$, while the CVN with $\alpha=1$ survives. Consequently, the existence of a maximum in $\mathcal{T}(\tau)$ results from the specific relation between $Q$ and $\tau$ and also is not of a resonance nature.

Very similar features are seen in the graphs of the standard deviation of the escape time $\Delta\mathcal{T}$. For given $\alpha$ the minima and maxima appear in the same places as for $\mathcal{T}(\tau)$. Furthermore, the maxima are much more distinct here. Comparing the whole curves $\mathcal{T}(\tau)$ and $\Delta\mathcal{T}(\tau)$ one can distinguish two regions. For $\tau$ smaller than 10 for any given $\alpha$ the curve $\Delta\mathcal{T}(\tau)$ follows $\mathcal{T}(\tau)$ almost exactly. Thus in this region the rate concept applies and the escape process can be characterized by a decay rate equal to the inverse of $\mathcal{T}(\tau)$ [4]. For greater $\tau$, however, $\Delta\mathcal{T}(\tau)$ exceeds $\mathcal{T}(\tau)$. Since for larger $\alpha$ the noise $z(t)$ vanishes more gradually as $\tau\to\infty$, the larger $\alpha$ is, the greater becomes the difference between $\Delta\mathcal{T}(\tau)$ and $\mathcal{T}(\tau)$. This reflects the fact that for large $\tau$, especially when it is much larger than the MFPT, the potential remains almost static during any escape attempt and the problem may be treated as an escape over an ensemble of static barriers with randomly distributed heights (the adiabatic approximation). The exponential dependence of the escape time on $\Delta U$ causes higher barriers to dominate in the averaged expressions. Consequently, the MFPT is greater than for the static barrier [8]. Also, $\Delta\mathcal{T}$ exceeds $\mathcal{T}$ and, if a maximum exists, it is better seen for $\Delta\mathcal{T}(\tau)$ than for $\mathcal{T}(\tau)$.

### B. Position of the barrier

Parts (b) of Figs. 1–5 show the results of measurements of $z_{\mathrm{top}}$, i.e., the value of the colored noise $-z(t)$ at the moment when the system variable $x(t)$ crosses the top of the barrier. This relates to the configuration of the potential during the escape event. In the figures we display its mean value $Z(\tau)$ [12] as well as its standard deviation $\Delta Z(\tau)$. For comparison, the standard deviation $S_D=\sqrt{D}$ of $z(t)$ is also shown.

The most important observation is that in any case, for CSN also, $Z(\tau)$ exhibits a maximum. It is located in the region between $\tau\approx0.3$ for $\alpha=0$ and $\tau\approx5$ for $\alpha=1.0$, i.e., for $\tau\sim O(1)$. These maxima mean that, regardless of the type of noise, for $\tau$ of the order of unity the system prefers to escape when the barrier is in its lower position. For smaller or larger values of $\tau$ escape events over higher barriers are relatively more probable. The region of $\tau$ for the occurrence of this maximum is limited so one can ask whether this effect is of a resonance nature. We will return to this question shortly.

Quite different is the dependence of $\Delta Z$ on $\tau$. For any value of $\alpha$ it almost equals $S_D$. A small deviation from this rule is noticeable only for large $\tau$ where $\Delta Z(\tau)$ falls slightly below $S_D$. Thus, $z_{\mathrm{top}}$ is a random variable with the same standard deviation as that of the process $z(t)$, but with a nonzero mean $Z(\tau)$.

For CSN the maximum of $Z(\tau)$ lies under the line $S_D(\tau)$. With increasing $\alpha$, the maximum moves toward this line, eventually just crossing it for the CVN case. In order to explain this distribution, note that for $\tau=1$ (the maximum appears for $\tau$ of the order of unity) the standard deviation of $z(t)$ has the same value $\sqrt{Q_0}$ for any $\alpha$. Thus if $\tau$ is slightly smaller the amplitude of the fluctuations for smaller $\alpha$ is larger. In contrast, for $\tau>1$ the larger $\alpha$ becomes the larger is the amplitude of the fluctuations.

The different rate of increase or decrease of the fluctuation amplitude with variation of $\tau$ for different $\alpha$ blurs the essence of this effect, however. In order to eliminate it we must consider the relative rather than the absolute height of the lowered barrier. The word ''relative'' means with respect to the actual possibilities, i.e., with respect to the amplitude of the barrier fluctuations for a given $\tau$. Such an approach seems obvious on looking, e.g., at Fig. 1(b). At $\tau\approx2$, where $Z(\tau)\approx\sqrt{Q/\tau}$, in order to escape over the barrier the system exploits much more the modulation of the barrier caused by the colored noise than it does at $\tau\approx0.3$, where, although the maximum of $Z(\tau)$ appears, the possibilities are greater [$Z(\tau)$ is only about $(1/2)S_D$]. In Fig. 6 we display the relative mean value of $z_{\mathrm{top}}$ defined as follows:

$$\tilde{Z}(\tau)=Z(\tau)/\sqrt{Q/\tau}. \tag{9}$$

The plots for different $\alpha$ differ only slightly. The maxima are distributed within a very small interval, 2–4.6. Their heights are almost the same and of the order of unity. The plots are shifted slightly toward the right as $\alpha$ increases.

After thus reducing the influence of the $\tau$ dependence of the $S_D$ of the OUN on the barrier fluctuation amplitude, we may suppose that the occurrence of a maximum of $\tilde{Z}(\tau)$ is of resonance origin only. It appears for $\tau$ of the order of a few units and this is the time scale of relaxation in the system. To

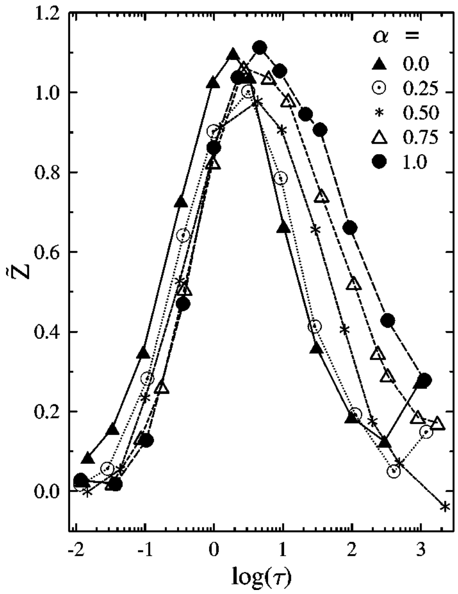


FIG. 6. Relative lowering of the barrier $\tilde{Z}(\tau)$ [Eq. (9)] at the moment of crossing over the top of the barrier for $\alpha=0$, 0.25, 0.50, 0.75, and 1.0. The other parameters as in Fig. 1.

understand this relation, let us recall that for a small noise in the standard (static) Kramers problem the system fluctuates for a very long time at the bottom of the well, waiting for a large enough fluctuation of the white noise $\xi(t)$ to kick it over the top of the barrier. Because $\xi(t)$ is a Gaussian process, the waiting time depends exponentially on the height of the barrier, and hence a lower barrier is greatly to be preferred. When this large fluctuation ultimately happens, it should persist for a duration at least of the order of the relaxation time $t_r$ of the system [13,11,4,5], which assures that the system has a long enough time to cross to the other side of the barrier. If the barrier rises during this stage, the system may return back to its initial well, thus increasing the waiting time. This explains why, in order to ensure the minimal escape time, the variation of the barrier, measured by the value of the correlation time $\tau$, should occur on a time scale longer than $t_r$. However, when $\tau$ becomes too long, there will also be enough time for a successful escape attempt over higher barrier configurations. This results in an increase of the mean height of the barrier at the moment of escape. Consequently, for $\tau \sim t_r$ a minimum appears in $z(t)$.

Following this discussion we may explain also the dependence on $\alpha$ of the plots in Fig. 6: as $\alpha$ decreases the amplitude of the barrier fluctuations increases, and so lower barriers can appear. This implies that a shorter time, albeit still of the order of $t_r$, will be required to cross to the other side of the barrier. Thus the resonant value of $\tau$ decreases.

## V. CONCLUSIONS

In this paper we have reported the results of our electronic analog experiments on the problem of an escape over a fluctuating barrier of potential. The potential fluctuation were caused by a few types of OUN with different relationships between the two parameters $\tau$ and $Q$. We measured the MFPT $\mathcal{T}$ and its standard deviation $\Delta\mathcal{T}$ for the threshold located at the top of the barrier. We also collected the mean value $Z$ and standard deviation $\Delta Z$ of the value of colored noise $z(t)$ at the moment of crossing the threshold.

Our main conclusion is that the resonance in the problem does not relate to the duration of correlation $\tau$ of the barrier noise and the escape time, as often believed when considering resonant activation problems. The resonance occurs between $\tau$ and the small part of the escape time during which the system jumps from the region of the potential well to the other side of the barrier. Since this time is of the order of the relaxation time of the system $t_r$ the resonance condition briefly reads

$$\tau \sim t_r . \tag{10}$$

In the resonance the system maximally exploits the stochastic lowering of the barrier by $z(t)$—an escape event typically happens through a relatively lower barrier.

This resonance may give rise to a minimum in $\mathcal{T}(\tau)$, known as resonant activation. But the resonance identified by us occurs also for CSN. In this case $\mathcal{T}(\tau)$ does not hit any minimum, implying that there is apparently no resonance for this noise. However, as we have shown, the dependence of the MFPT on $\tau$ arises because of the dependence on $\tau$ of two factors: the mean relative height of the barrier $\tilde{Z}$ during the escape event, and the standard deviation $S_D$ of the barrier noise. For CSN the decrease of $S_D$ is stronger than the increase of $\tilde{Z}$ and consequently $\mathcal{T}(\tau)$ increases monotonically beyond the resonant region Eq. (10).

In the region of large $\tau$ a maximum of $\mathcal{T}(\tau)$ can appear, known as an inhibition of activation [8]. Since the nature of this feature was not clearly identified it was not referred to as a resonance in [8]. According to the present analysis, and exploiting similarities between the two limits of $\tau$ ($\tau \to 0$ and $\tau \to \infty$) discussed in [8], we can state that the appearance of this maximum is not, in fact, of a resonance character. We have not identified a corresponding time scale in the system. Thus the inhibition of activation appears only as a consequence of the dependence of $Q$ on $\tau$.

We believe that our findings are general in the sense that they do not depend on a specific definition of the escape time. Here we characterize it by means of the MFPT; we note that other possibilities exist, e.g., the Kramers flux-over-population rate [14] or the lowest nonzero eigenvalue [15]. Our conviction is especially supported by the very recent paper of Reimann *et al.* [16] proving the equivalence of the flux-over-population rate with the inverse of the MFPT.

## ACKNOWLEDGMENTS

This research was supported by the Engineering and Physical Sciences Research Council under Grants Nos. GR/L69381 and GR/L38875. J.I. and I.K.K. are very grateful to Lancaster University for hospitality during their visit, when most of the work was completed.

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